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## INTERACTIVE OPTIMIZATION OF INTERNAL COMBUSTION ENGINE TESTS BY MEANS OF SEQUENTIAL EXPERIMENTAL DESIGN

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### ABSTRACT

The application of a Sequential Experimental Design technique to the design of experiments for Internal Combustion Engines is presented, in order to maximize the information achieved in experimental tests by guiding them in structured way.

This technique can be used whenever the parameters of a linear or non linear model have to be estimated from experimental data, or also from the results of a computational code. The new experimental condition is determined interactively by minimizing the expected volume of the model parameter confidence region.

The method has been applied to the estimation of a black-box model for the specific consumption of a spark ignition engine as a function of four operating variables, used for engine control applications. The proposed experimental strategy has been compared with conventional ones, for a data set of more than 1000 experimental conditions, and a more rapid convergence to the reference values of model parameters has been observed.

The results suggest that substantial benefits can be achieved in reducing and guiding the experimental tests for the study and the design of Internal Combustion Engines, for which extensive investigations are often required.

### MODELS AND EXPERIMENTAL DESIGN

Modern approach to the study and the design of internal combustion engines (ICE) is based both on experimental analysis and modelling techniques. Notwithstanding the increasing role played by these latter in recent years, it is common opinion that the complete description of ICE by means of exclusive recourse to modelling techniques is still far from the possibility, at least in next future.

On the other hand, computational power required to complete predictive models, even available, could in many case conflict with the

available resources. In fact, the choice of the most suitable model is derived by the analysis of different factors, such as i) state-of-art of theoretical knowledge in the given sector, ii) possibility and cost of experimental investigations and, last but not least, iii) objectives and operative context of the work. A pictorial scheme of the processes involved in the phases of development and validation of a mathematical model is reported in fig.1 [Spriet and Vansteenkiste, 1988]. According to the recourse to *a priori* knowledge, to experimental data or to an appropriate mixing of them, the models are respectively named as *white-box*, *black-box* or *grey-box* [Spriet and Vansteenkiste, 1988]. Therefore, even when predictive *white-box* models are available, the analysis of such criteria in the given context could suggest the use of *grey-box* or *black-box* models. In such cases, an experimental analysis is needed in order to provide to the model the missed physical information. Therefore, whenever experimental conditions can be determined by the researcher and significant resources are involved in the experimental phase, the maximization of information obtained by this latter represent a significant target, which is the objective of the Experimental Design (ED) techniques. Their interactions with the phases of model development and validation are indicated in fig.1.

These considerations largely apply to R&D and design of internal combustion engines, which, also due to the unavailability of fully predictive models, are mostly based on extensive experimental analysis. In many cases, a complete system characterization would require a high number of experimental tests, due to the relevant number of independent variables, non linearity and presence of uncertainties, turbulence and other stochastic effects. For instance, the engine mapping procedures usually adopted for control strategy optimization require the experimental analysis over a number of engine conditions of the order of thousands [Sweet, 1981, Tennant et al., 1979, Rizzo, 1987, Rizzo and Pianese, 1989, Kormanski and

Rudzinska, 1994], which results in considerable time and costs and also in significant deterioration in the investigated system. Many other examples could be quoted where engine model parameters are estimated by experimental data, even within the Authors experience only: parameter estimation for turbulent combustion models [Bozza et al., 1991], quasi-equilibrium models for *on-board* estimation of NO<sub>x</sub> emissions from pressure cycles [De Petris et al., 1989], two phases transient flow characterization in the intake manifold of injection engines [Gambino et al., 1994]. It clearly emerges the exigency of methods for a proper and rationale choice of the experiments.

These considerations can be also extended to the cases where the experiments, within a hierarchical modelling approach, are performed by means of mathematical models with high computational cost. An example on the use of *Experimental Design Theory* (EDT) within a problem of structural optimization to guide the numerical experiments performed by Finite Elements Method FEM is reported by van Campen et al. [1990].

#### METHODS OF NON LINEAR EXPERIMENTAL DESIGN

The objective of EDT is the design of the optimal experimental plan, which consists in the determination of the vector of independent variables, the number of the test to be performed in each condition and the sequence of the tests, in order to maximize the information obtained from the experiments [Law and Kelton, 1991, Draper and

Smith, 1980, van Campen et al., 1990]. A wide review on the applications of EDT to non linear problems has been presented by Ford et al. [1990]. Regard to the applications to engineering found in the open literature, most of them lie to chemical engineering. Little attention has been paid to the applications of EDT to internal combustion engines (ICE), in spite of the relevant experimental resources often involved in this field.

A synthetic review of some classical ED problems is presented in the following. A general non linear model is considered:

$$\mathbf{Y} = f(\mathbf{X}, \boldsymbol{\beta}) + \varepsilon \quad (1)$$

The experimental plane is characterized by n observations, and by a distribution of vector of independent variables ( $\mathbf{X}_i$ ,  $i=1,n$ ). In case that repeated observations are allowed, two or more values can coincide. For a given experimental plane, the optimal values of model parameters  $\boldsymbol{\beta}$  can be found by solving a problem of non linear regression, with least squares techniques:

$$\min_{\boldsymbol{\beta}} \left( \sum_{i=1}^n [Y_i^* - Y_i(\mathbf{X}_i, \boldsymbol{\beta})]^2 \right) \quad (2)$$

Confidence intervals of the parameters  $\boldsymbol{\beta}$  in case of non linear models can be also estimated [Draper and Smith, 1981, Ratkowsky, 1983, Pianese and Rizzo, 1995].

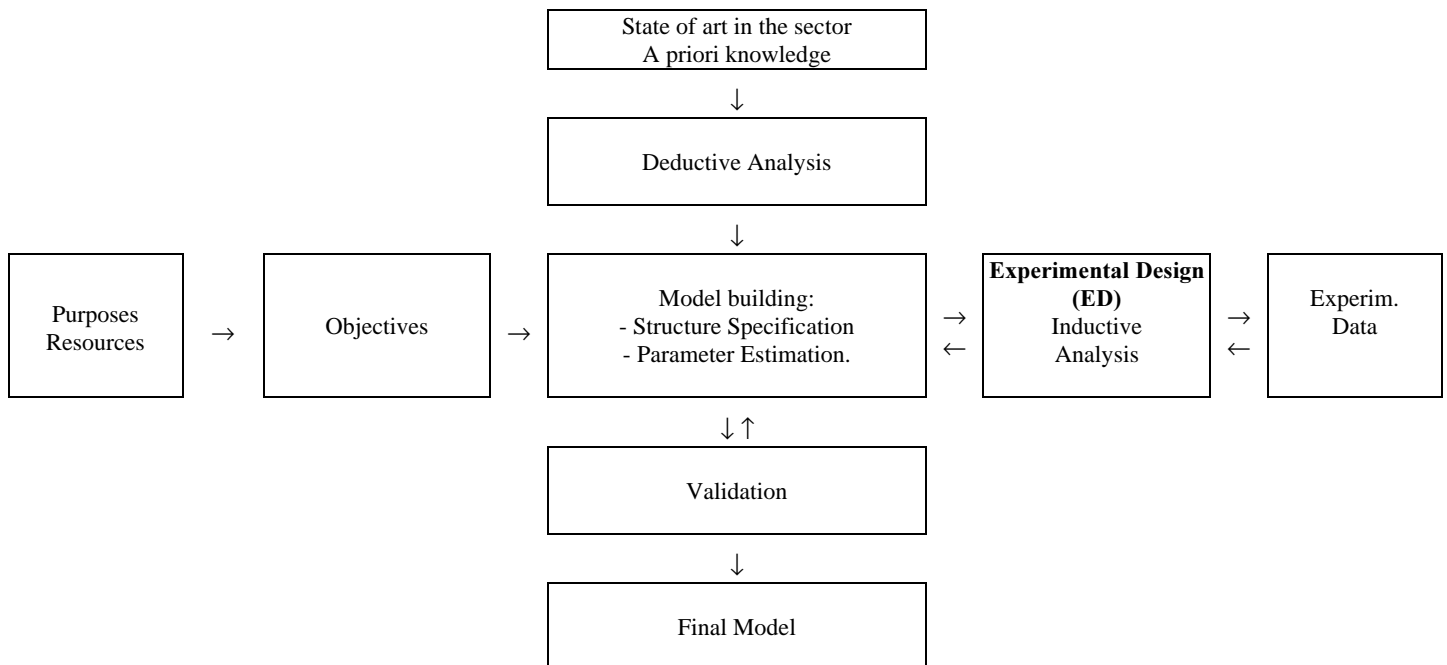


Fig.1 - Scheme of the modelling process and role of Experimental Design.

The solutions determined by EDT can result in not obvious distributions of independent variables X, even in the simplest cases. For instance, in case that a least square straight line has to be determined by distributing n observations in a given interval ( $X_{\min}$ - $X_{\max}$ ), with constant and known variance of independent variable Y (fig. 2), the best choice is represented by case c, with repeated tests at the two ends of the interval [Draper and Smith, 1980].

ED methods can be divided in:

- static, when the sequence of independent variables X is predetermined;
- sequential, when the next choice of X is performed starting on previous values of X and Y, according to given criteria.

Sequential Design methods can be further divided in Fully Sequential Design and Batch Sequential Design, according as one or more values of X are determined starting from previous results.

The optimal distribution of X depends on function structure and, in case of non linear models, by true value of the parameters  $\beta$ , which is in general unknown. With static ED methods optimal distributions are determined starting from an estimate of parameters, while by means of sequential methods solution can avail of their recursive updating. Thus, application of sequential ED methods require on-line data processing, which, at the cost of increased complexity, allows a continuous monitoring of the experimental tests and a more precise control about the achievement of the targets.

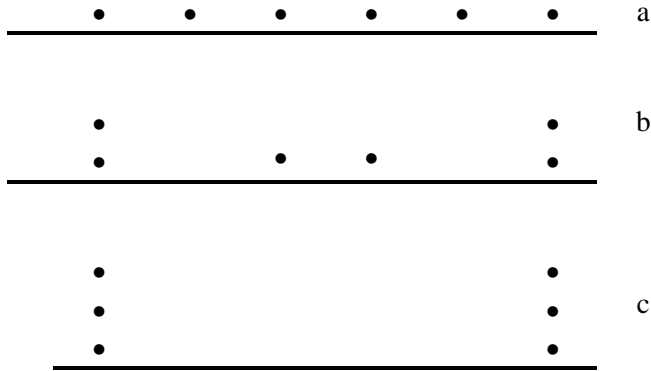


Fig.2 - Possible distributions of experimental conditions for the estimation of a linear regression, for a given number of observations (n=6).

Most of the criteria adopted to determine the optimal ED refer to the variance of the parameters of the model, which is the more well defined the less is the volume of confidence region of its parameters. This volume is a function of the following matrix:

$$M = \left( ZZ^T \right)^{-1} \quad (3)$$

where Z is the sensitivity matrix:

$$Z_{i,j} = \frac{\partial Y_i}{\partial \beta_j} \quad (4)$$

Optimal conditions can be found according to different criteria:

1. D-Optimality: minimize  $D_1 = \left| \left( ZZ^T \right)^{-1} \right|$ , or maximize  $D = \left| \left( ZZ^T \right) \right|$ .
2. G-Optimality: minimize maximum variance.
3. V-Optimality: minimize average variance.

In this paper the first criterion has been adopted, which can be drawn to the solution of the problem:

$$\max_{X_i} D \quad (5)$$

where D is the following determinant:

$$D = \left| \left( ZZ^T \right) \right| \quad (6)$$

This criterion is best adopted when the major target is model parameter estimation, with high number of variables and/or levels, and with multiple response vector Y [van Campen et al., 1990].

In case of a simple linear homogeneous model  $Y = \beta X$  with one variable (p=1) and one observation (n=1), by equations (4) and (6) it follows  $Z_{1,1} = X$  and  $D = X^2$ . The optimal solution, corresponding to the maximization of D with (5), is achieved by locating X as far as possible from the origin. It is evident that, for given uncertainty on Y values, this choice allows the maximum precision in determining the slope  $\beta$ .

An FSD (Fully Sequential Design) procedure can be therefore defined:

1. Initial selection of n observations  $X_i$ .
2. Estimation of model parameters  $\beta$  (2) for the current set of n observations.
3. Possible evaluation of confidence intervals for  $\beta$ .
4. Check of termination criteria (maximum number of observations or model parameter convergence).
5. Numerical evaluation of the sensitivity matrix Z (4).
6. Selection of the next (n+1)-th observation X which maximize the determinant D (5).
7. Updating of number of observations n and return to point 3.

Determinant D is computed by multiplying the diagonal terms resulting from a LU decomposition of the original matrix [Press et al., 1992]. Logarithmic transforms have been adopted in order to avoid underflow or overflow occurrence, in case of operation with matrices of significant size. In the present case, where the set of independent variables X was predetermined, the choice of the next condition in the phase 6 has been performed by exhaustive evaluation over all the X values not yet included in the experimental plane. For each value of X, the last row of sensitivity matrix (4) and the determinant (6) were

computed. In case where the set of X is very large, or where it is not predetermined, continuous or discrete optimization techniques can be adopted [van Campen et al., 1990].

This procedure has been first tested for a linear model of one independent variable X:

$$Y = \beta_1 + \beta_2 \quad (7)$$

The observed values  $Y^*$  have been generated by the following function:

$$Y^* = 1 + 2X + \varepsilon \quad (8)$$

where  $\varepsilon$  represents a random variable with uniform distribution in the interval (-0.5+0.5). The variable X can assume 11 discrete values between 0 and 1:

$$X = (0, .1, .2, .3, .4, .5, .6, .7, .8, .9, 1) \quad (9)$$

The sequential ED method has been compared with 5 different strategies based on random choice of the values of X. Number of observations has been increased from 4 to 25, starting from the same initial distributions of X. The estimated values of the two coefficients of (7) versus observations are plotted in figs. 3a-3b. It can be noticed that the ED method exhibits faster convergence to the true values of  $\beta$  (8). This result has been obtained selecting only X values placed at the two ends of the interval (0-1), in accordance with the result stated by classical EDT. With the random strategies, almost uniform distributions of X values on the interval (0-1) resulted.

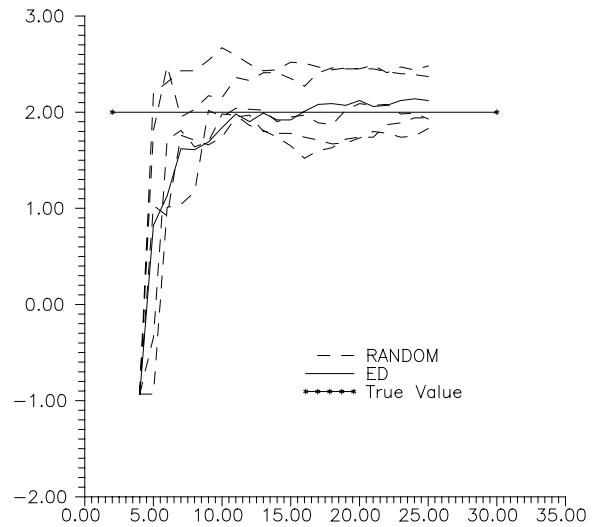


Fig.3b - Comparison between ED method and random strategies - Estimate of parameter  $\beta_2$  versus number of observations.

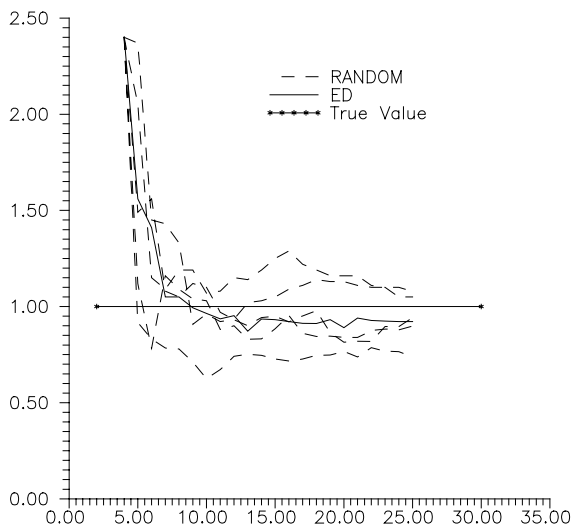


Fig.3a - Comparison between ED method and random strategies - Estimate of parameter  $\beta_1$  versus number of observations.

#### APPLICATION OF AN ED TECHNIQUE TO THE ESTIMATION OF A MODEL FOR FUEL CONSUMPTION

In order to check the benefits obtainable by using the ED technique in case of more complex functions, representative of real problems in engine field, an application to a model for estimation of fuel consumption as a function of four operative variables has been considered. A set of steady state experimental data measured on an Alfa Romeo AR2000 engine at Laboratories of Istituto Motori CNR has been utilized. These data have been used for the development of engine dynamic models [Pianese and Rizzo, 1992, Gambino et al., 1994, Siviero et al, 1995] and engine optimal control [Abaffy et al., 1987, Rizzo and Pianese, 1989, Pianese and Rizzo, 1992, Arsie et al., 1996].

The whole sample of experimental data consists of 1043 different operating conditions, without repeated measurements. The ranges of variation of the four operating variables (torque, rotational speed, air-fuel ratio and spark advance) are described in tab.I. For each variable, the entire range has been divided in four sub-ranges.

|           | X <sub>1</sub><br>Torque<br>[Nm] | X <sub>2</sub><br>Speed<br>[rpm] | X <sub>3</sub><br>A/F ratio<br>[l] | X <sub>4</sub><br>Sp. adv.<br>[deg] |
|-----------|----------------------------------|----------------------------------|------------------------------------|-------------------------------------|
| sub-range | 20 - 80                          | 1000-1750                        | 10.0 - 17.0                        | 2 - 45                              |
| 1st       | 20 - 35                          | 1000-1187                        | 10.0 - 11.7                        | 2 - 13                              |
| 2nd       | 35 - 50                          | 1187-1375                        | 11.7 - 13.5                        | 13 - 23                             |

|     |         |           |             |         |
|-----|---------|-----------|-------------|---------|
| 3rd | 50 - 65 | 1375-1562 | 13.5 - 15.2 | 23 - 34 |
| 4th | 65 - 80 | 1562-1750 | 15.2 - 17.0 | 34 - 45 |

Tab. I - Ranges of variation for the independent variables.

Specific fuel consumption has been expressed as polynomial function of the four variables  $X_1$ ,  $X_2$ ,  $X_3$  and  $X_4$ :

$$y = \sum_{i=1}^n k_i x_1^{\alpha_{i,1}} x_2^{\alpha_{i,2}} x_3^{\alpha_{i,3}} x_4^{\alpha_{i,4}} \quad (10)$$

The exponents referring to the 13 terms considered in (10) are reported in tab.II.

The coefficients  $\beta_1$ - $\beta_{13}$  of (10) have been computed by least square technique, over the whole sample. The optimal values of the coefficients are reported in tab.II. The corresponding confidence intervals at the probability level  $\alpha$  have been estimated by numerical evaluation of the confidence contours of the function sum of squares (11) [Draper and Smith, 1981, Ratkowsky, 1983, Pianese and Rizzo, 1995]:

$$S(\beta) = S(\beta^*) \left[ 1 + \frac{p}{n-p} F(p, n-p, 1-\alpha) \right] \quad (11)$$

The comparison between computed and measured data on the whole set of experimental data is presented in fig.4b ( $R^2=0.87$ ). This model, and the corresponding values of their parameters, has been considered as reference point for the subsequent computations. In this case, a single regression model in four variables has been considered

(global model). In terms of fitting accuracy, better results have been obtained by joint use of regression models (local models) in two or three variables for each operating point (torque, rpm) and of multi-dimensional interpolation techniques [Kormanski and Rudzinska, 1994, Sweet, 1981]. For the present application, where the major target is the comparison of experimental strategies rather than the achievement of the highest fitting accuracy, a global model has been preferred because it allows a more compact mathematical representation.

The results obtained by the ED method have been compared with the ones corresponding to 5 different strategies, based on random choices of the independent variables X. The number of observations has been varied from 15 to 100. In all the cases equal starting distribution of X (n=15) have been selected, corresponding to a rather uniform distribution of the four variables over their ranges. For each experimental plane, the optimal values of parameters  $\beta$  and an error term E, expressing the differences between the actual values of parameters and their reference values  $\beta^*$  (tab.II), have been computed:

$$E = \frac{\sqrt{\sum_{i=1}^n (\beta_i - \beta_i^*)^2}}{n} \quad (12)$$

The error (12) versus the number of observations is plotted in fig.4a, for the different strategies. It can be noticed that for the ED technique the convergence of model parameters to the reference values is significantly faster. Therefore, a given model precision can be achieved with reduced experimental work, or, conversely, higher model accuracy can be obtained with a given number of observations.

| Coeff. | $\alpha_1$<br>Torque | $\alpha_2$<br>rpm | $\alpha_3$<br>A/F | $\alpha_4$<br>Spark<br>ad. | Optimal<br>value $\beta^*$ | 95%<br>confidence interval |          |
|--------|----------------------|-------------------|-------------------|----------------------------|----------------------------|----------------------------|----------|
|        |                      |                   |                   |                            |                            |                            |          |
| 1      | 0                    | 0                 | 0                 | 0                          | 1.9531                     | 1.9515                     | 1.9547   |
| 2      | 0                    | 0                 | 0                 | 1                          | -.83731                    | -.85592                    | -.8187   |
| 3      | 0                    | 0                 | 1                 | 0                          | -2.4948                    | -2.4968                    | -2.4927  |
| 4      | 0                    | 1                 | 0                 | 0                          | .099939                    | .097718                    | .10216   |
| 5      | 1                    | 0                 | 0                 | 0                          | -.048702                   | -.049062                   | -.048341 |
| 6      | 1                    | 1                 | 0                 | 0                          | -.019438                   | -.01987                    | -.019006 |
| 7      | 1                    | 0                 | 1                 | 0                          | -.015223                   | -.015562                   | -.014885 |
| 8      | 1                    | 0                 | 0                 | 1                          | .11555                     | .11298                     | .11811   |
| 9      | 0                    | 1                 | 1                 | 0                          | .092132                    | .090084                    | .094179  |
| 10     | 0                    | 1                 | 0                 | 1                          | -.20392                    | -.21752                    | -.19033  |
| 11     | 0                    | 0                 | 1                 | 1                          | -.67298                    | -.68794                    | -.65803  |
| 12     | 0                    | 0                 | 2                 | 0                          | 1.3132                     | 1.31                       | 1.3165   |
| 13     | 0                    | 0                 | 0                 | 2                          | 1.5198                     | 1.4861                     | 1.5536   |

Tab.II - Polynomial model for Specific Fuel Consumption. Exponents of independent variables

## Optimal values and confidence intervals for the model parameters

In order to characterize the different experimental strategies, for each independent variables the operating range has been divided in four sub-ranges (tab.I), and the 6 possible combinations between the couples of 4 variables have been represented in the tables IIIa-c. For instance, table IIIa, referring to the whole sample of 1043 observations, indicates that 69 points belong to the first sub-range of rpm and to the first sub-range of torque, and so on. For each 4x4 sub-table corresponding to a couple of variables, the sum of the terms equals the number of observations in the given sample.

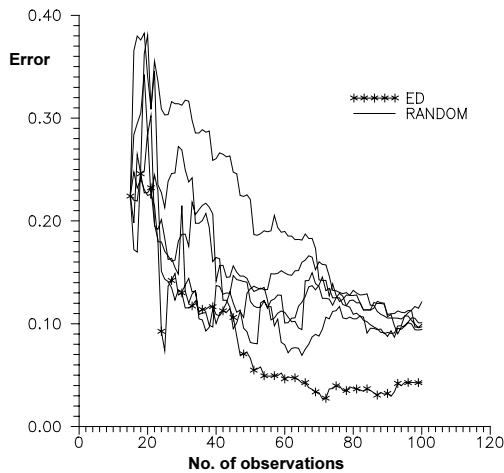


Fig.4a - Error on model coefficients (12) versus observations for the different methods.

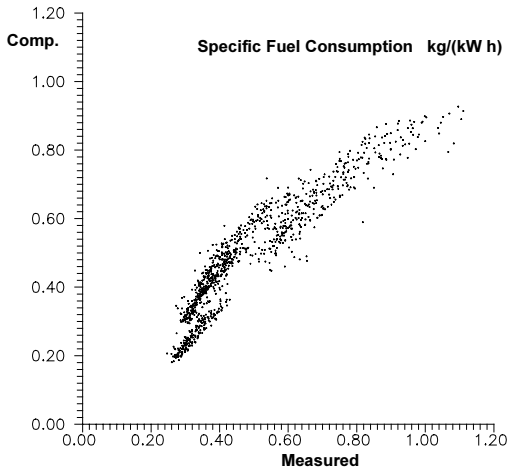


Fig.4b - Comparison between measured and computed values of specific fuel consumption, using the whole set of data (n=1043).

It can be noticed (tab.IIIa) that the whole set of experimental data exhibits a quite uniform distribution for torque, rpm and spark advance, while most of air-fuel ratio values are located around the stoichiometric value ( $\approx 15$ ), with lower density in first and fourth sub-ranges, corresponding to very rich and very lean mixtures.

In the table IIIb the distributions of experimental points selected by the ED method (n=100) are described. For each combination of variables, the percent ratio between the number of data selected by this method and the number of observations in the whole data set is reported. It can be noticed that most of the experimental conditions selected by the ED method belong to the corners of each 4x4 sub-table and are usually mostly concentrated in the first and fourth rows and columns and less dense in the central regions. The tendency toward concentration in the corners is particularly marked for those variables which show a linear dependence with the response variable (10), as torque and rotational speed (tab.II). In the cases where at least one of the variables has a non linear dependence, as air-fuel ratio and spark advance, the observations are less concentrated in the corners and partly distributed along the boundaries. The last table (tab.IIIc) refers to the results obtained with one of the random methods. In this case the distributions are more uniform, and tend to reflect the distribution in the whole data set. Similar results have been also obtained with the other random cases.

## CONCLUSIONS AND FUTURE WORK

A technique of Sequential Experimental Design has been presented, which allows to interactively determine the experimental conditions corresponding to the minimum value of the confidence region for the parameters of a linear or non-linear model. The method has been applied to the estimation of a polynomial regression model for the specific fuel consumption of a spark ignition engine as a function of four independent variables. The test, carried out over a data set of more than 1000 experimental points, has shown that faster convergence to the reference values of model parameters can be achieved with respect to alternative methods. Therefore, given model precision can be achieved with reduced experimental work, or, conversely, higher model accuracy can be obtained with a given number of observations. The results suggest that an effective contribute to the reduction and rationalization of the extensive experimental plans often occurring in the design and testing of internal combustion engines can be offered by ED techniques.

Further work is in progress in order to extend this method to the case where multiple responses have to be estimated (e.g. fuel consumption and emissions) and to its applications to more complex cases, as the estimation of heat release and other engine parameters by means of thermodynamic models.

| Rotational Speed |    |    |    | Air/Fuel Ratio |     |     |    | Spark Advance |    |    |    | Torque            |
|------------------|----|----|----|----------------|-----|-----|----|---------------|----|----|----|-------------------|
| 69               | 73 | 73 | 93 | 34             | 125 | 119 | 30 | 108           | 64 | 66 | 70 |                   |
| 73               | 59 | 73 | 68 | 15             | 105 | 122 | 31 | 93            | 60 | 61 | 59 |                   |
| 66               | 69 | 66 | 58 | 5              | 100 | 114 | 40 | 93            | 55 | 57 | 54 |                   |
| 44               | 51 | 56 | 52 | 6              | 90  | 89  | 18 | 77            | 51 | 46 | 29 |                   |
|                  |    |    |    | 21             | 103 | 94  | 34 | 95            | 48 | 57 | 52 | Rotat.<br>Speed   |
|                  |    |    |    | 15             | 104 | 102 | 31 | 84            | 65 | 51 | 52 |                   |
|                  |    |    |    | 13             | 106 | 125 | 24 | 101           | 55 | 58 | 54 |                   |
|                  |    |    |    | 11             | 107 | 123 | 30 | 91            | 62 | 64 | 54 | Air/Fuel<br>Ratio |
|                  |    |    |    | 15             | 19  | 9   | 17 |               |    |    |    |                   |
|                  |    |    |    | 147            | 91  | 92  | 90 |               |    |    |    |                   |
|                  |    |    |    | 183            | 90  | 96  | 75 |               |    |    |    |                   |
|                  |    |    |    | 26             | 30  | 33  | 30 |               |    |    |    |                   |

Tab.IIIa - Distribution of the experimental points in the whole data set (n=1043).

| Rotational Speed |   |   |    | Air/Fuel Ratio |    |    |    | Spark Advance |   |    |    | Torque            |
|------------------|---|---|----|----------------|----|----|----|---------------|---|----|----|-------------------|
| 25               | 4 | 3 | 23 | 32             | 8  | 8  | 40 | 16            | 5 | 6  | 27 |                   |
| 4                | 2 | 4 | 1  | 20             | 2  | 1  | 6  | 3             | 5 | 0  | 3  |                   |
| 11               | 4 | 3 | 3  | 0              | 1  | 6  | 15 | 3             | 2 | 2  | 17 |                   |
| 27               | 6 | 4 | 35 | 67             | 16 | 11 | 39 | 19            | 8 | 11 | 38 |                   |
|                  |   |   |    | 38             | 8  | 13 | 32 | 19            | 4 | 9  | 27 | Rotat.<br>Speed   |
|                  |   |   |    | 13             | 3  | 3  | 6  | 2             | 3 | 0  | 12 |                   |
|                  |   |   |    | 23             | 1  | 2  | 8  | 1             | 4 | 3  | 7  |                   |
|                  |   |   |    | 45             | 14 | 8  | 40 | 19            | 8 | 5  | 31 | Air/Fuel<br>Ratio |
|                  |   |   |    | 40             | 11 | 11 | 53 |               |   |    |    |                   |
|                  |   |   |    | 7              | 3  | 2  | 12 |               |   |    |    |                   |
|                  |   |   |    | 8              | 2  | 3  | 11 |               |   |    |    |                   |
|                  |   |   |    | 23             | 13 | 12 | 43 |               |   |    |    |                   |

Tab.IIIb - Distribution of experimental points with the ED method (n=100)

Ratio (%) between the number of observations in each sub-range and the ones in the whole data set (tab.IIIa).

| Rotational Speed |    |    |    | Air/Fuel Ratio |    |    |    | Spark Advance |    |    |    | Torque            |
|------------------|----|----|----|----------------|----|----|----|---------------|----|----|----|-------------------|
| 10               | 10 | 10 | 10 | 29             | 4  | 9  | 13 | 13            | 11 | 11 | 3  |                   |
| 8                | 12 | 7  | 13 | 40             | 7  | 8  | 13 | 6             | 13 | 15 | 7  |                   |
| 6                | 14 | 12 | 2  | 20             | 3  | 9  | 23 | 10            | 9  | 5  | 11 |                   |
| 11               | 18 | 2  | 10 | 33             | 7  | 9  | 22 | 8             | 10 | 15 | 7  |                   |
|                  |    |    |    | 24             | 4  | 7  | 18 | 11            | 4  | 11 | 8  | Rotat.<br>Speed   |
|                  |    |    |    | 27             | 9  | 11 | 29 | 12            | 18 | 12 | 10 |                   |
|                  |    |    |    | 38             | 3  | 6  | 21 | 7             | 11 | 9  | 6  |                   |
|                  |    |    |    | 45             | 5  | 11 | 3  | 9             | 8  | 14 | 4  | Air/Fuel<br>Ratio |
|                  |    |    |    | 53             | 32 | 33 | 12 |               |    |    |    |                   |
|                  |    |    |    | 3              | 8  | 7  | 3  |               |    |    |    |                   |
|                  |    |    |    | 9              | 9  | 9  | 8  |               |    |    |    |                   |
|                  |    |    |    | 23             | 13 | 24 | 10 |               |    |    |    |                   |

Tab.IIIc - Distribution of experimental points with a random method (n=100)

Ratio (%) between the number of observations in each sub-range and the ones in the whole data set (tab.IIIa).

## NOMENCLATURE

|                 |   |
|-----------------|---|
| $\alpha$        | Exponents of the independent variables                      |
| $\beta$         | Model parameters  |
| $\beta^*$       | Estimate of model parameters obtained with the whole sample |
| $\beta_{i,inf}$ | Lower limit of confidence interval for the i-th parameter   |
| $\beta_{i,sup}$ | Upper limit of confidence interval for the i-th parameter   |
| <b>SFC</b>      | Specific Fuel Consumption [kg/(kW h)]                       |
| D               | Determinant   |
| $\varepsilon$   | Error   |
| F               | F Distribution  |
| m               | Number of independent variables                             |
| n               | Number of observations                                      |
| p               | Number of parameters  |
| S               | Sum of squares  |
| S*              | Optimal value of sum of square                              |
| X               | Vector of the independent variables                         |
| Y               | Dependent or response variables                             |
| Y*              | Measured values of response variables                       |
| Z               | Sensitivity matrix (n x p)                                  |

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